GEORGIA TECH SCHOOL OF MATHEMATICS

MATH 1502

CALCULUS II

Final Exam 2 hours & 50 minutes

December 12th 2007

First Name : ____

Last Name : ____

1	
2	
3a-3b	
4a-4b-4c-4d	
5a-5b	
6	
7	
8a-8b-8c-8d	
9a-9b	
10a-10b-10c	
11a-11b-11c-11d-11e	
12a-12b-12c-12d	

WARNING :

Read carefully, read the comments in italic, take your time, do not panic and double check what you write.

Write the result cleanly and use the blank pages for your calculations.

Take the time to write in plain English the arguments used to get or justify the answer.

1. Give the value of $P^{(7)}(0)$ if

$$P(x) = 1 - 2x + 3x^2 - 5x^3 + 8x^5 - 13x^7 + 21x^{11} - 34x^{13}$$

$$P^{(7)}(0) \; =$$

2. Give the Taylor *series*, near $x = 0$ of

$$\cos 2x \ =$$

3. (a) Compute the limit (*Give explicitly the rule used to get the result*)

$$\lim_{x \to 0} \frac{\sin x}{x} =$$

(b) Is the following integral convergent ?

$$\int_0^1 \frac{dx}{x^{3/4}}$$

4. Tell whether the following series converge or not and indicate the test used to conclude

(a)

$$\sum_{n=1}^{\infty}(1+\frac{1}{n^2})$$

Converges ☐ *Diverges* ☐

Test used :

(b)

$$\sum_{n=0}^{\infty}\frac{(n+3)!}{2^n\,n!}$$

Converges ☐ *Diverges* ☐

Test used :

(Use this page for your calculations)

(c)

$$\sum_{n=0}^{\infty} \frac{1}{(1+3n^3)^{0.3}}$$

Converges ☐ *Diverges* ☐

Test used :

(d)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\ln(n^2+1)}$$

Converges ☐ *Diverges* ☐

Test used :

(Use this page for your calculations)

5. For any pair $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ of vectors in $\mathbb{R}^2$ let A, B be the matrices

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a_1 & 1 & 0 \\ a_2 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ b_1 & 1 & 0 \\ b_2 & 0 & 1 \end{bmatrix}$$

(a) Compute the product AB

$$AB = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}$$

(b) Show that A is invertible and compute its inverse
(*Hint : use the computation of* AB)

(Use this page for your calculations)

6. If $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ show that (*parallelogram identity*)

$$|\mathbf{x}+\mathbf{y}|^2 + |\mathbf{x}-\mathbf{y}|^2 = 2|\mathbf{x}|^2 + 2|\mathbf{y}|^2$$

7. Compute or draw the image of the unit square $[0,1] \times [0,1]$ by the matrix $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.

8. Consider the system of linear equations

$$\begin{aligned} x - 2y + az &= b \\ x + y + z &= 0 \\ 3y + z &= 2 \end{aligned}$$

For which values of a, b, if any, does this system have

(a) a unique solution ?

(b) give this solution

(c) no solution ?

(d) an infinite number of solutions ?

Unique solution $a\,,\ b\ =$

Solution $=$

No solution $a\,,\ b\ =$

$\infty\#$ solutions $a\,,\ b\ =$

(Use this page for your calculations)

(Use this page for your calculations)

9. Let A be the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

(a) find its determinant

(b) find its inverse

$$\det(A) \;=$$

$$A^{-1} \;= \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

(Use this page for your calculations)

(Use this page for your calculations)

10. Let A and $\mathbf{b} \in \mathbb{R}^3$ be given by

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(a) What is the rank of A?

(b) Find the vector $\mathbf{a} \in \mathrm{Im}(A)$ closest to $\mathbf{b}$

(c) Find the distance from $\mathbf{b}$ to $\mathrm{Im}(A)$

$\mathrm{rank}(A) =$

$\mathbf{a} =$

$\mathrm{dist}(\mathbf{b}, \mathrm{Im}(A)) =$

(Use this page for your calculations)

(Use this page for your calculations)

11. Consider the matrix $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 3 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 2 & 0 & 3 \end{bmatrix}$

   (a) Find a basis for $\operatorname{Im}(A)$
   (b) Find a basis for $\operatorname{Ker}(A)$
   (c) Find the orthogonal projection onto $\operatorname{Ker}(A^t)$
   (d) Find the orthogonal projection onto $\operatorname{Im}(A^t)$
   (e) Find an orthonormal basis for $\operatorname{Im}(A)$
   (*Hint : two vectors of the basis of* $\operatorname{Im}(A)$ *are already orthogonal*)

Basis for $\operatorname{Im}(A)$:

Basis for $\operatorname{Ker}(A)$:

Projection onto $\operatorname{Ker}(A^t)$:

Projection onto $\operatorname{Im}(A^t)$:

Orthonormal basis for $\operatorname{Im}(A)$:

(Use this page for your calculations)

(Use this page for your calculations)

12. Let $A = \begin{bmatrix} 3 & 4 \\ 1 & 0 \end{bmatrix}$

    (a) Find the trace of A

    (b) Find its eigenvalues

    (c) Find its eigenvectors

    (d) Write A as VDV^{-1} where D is a diagonal matrix : give both D and V

$\mathrm{tr}(A) \ =$

Eigenvalues of A :

Eigenvectors of A :

Diagonal form :

(Use this page for your calculations)

(Use this page for your calculations)