GEORGIA TECH SCHOOL OF MATHEMATICS

MATH 1502

CALCULUS II

Test # 2 : 50 minutes

November 5th 2007

First Name : _____

Last Name : _____

1	
2	
3	
4	
5a 5b 5c 5d	
6	
7	
8a 8b 8c	

WARNING :

Read carefully, read the comments in italic, take your time, do not panic and double check what you write.

Write the result cleanly and use the blank pages for your calculations.

Take the time to write in plain English the arguments used to get or justify the answer.

1. Find a matrix A such that

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} A = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \end{bmatrix}$$

(Use this page for your calculations)

2. Let $x = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$ and $y = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$ be two vectors in $\mathbb{R}^3$. Compute the cosine of their angle θ.

$$\cos\theta \ =$$

3. Compute or draw the image of the unit square $[0,1] \times [0,1]$ by the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

4. Find an equation for the plane containing the points $p_0 = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$, $p_1 = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$ and $p_2 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$.

Equation =

(Use this page for your calculations)

5. Consider the system of linear equations

$$\begin{aligned} x + y &= 1 \\ 2x + y + az &= 1 \\ 2x - z &= b \end{aligned}$$

For which values of a, b, if any, does this system have

(a) a unique solution ? Then give this solution

(b) no solution ?

(c) an infinite number of solution ?

Unique solution $a\,,\ b\ =$

Solution $=$

No solution $a\,,\ b\ =$

$\infty\#$ solutions $a\,,\ b\ =$

(Use this page for your calculations)

6. Compute the inverse (if any) of the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \end{bmatrix}$$

(Use this page for your calculations)

7. Compute the reduced row echelon form rref(A) of

$$A = \begin{bmatrix} 4 & 3 & 7 & 4 \\ -1 & 3 & 2 & -1 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

$$\text{rref}(A) = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

(Use this page for your calculations)

8. Let $A = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 1 & 2 & 3 & -2 & -1 \\ 3 & 0 & 5 & -4 & 1 \end{bmatrix}$. Then

   (a) What is the rank of A ?

   (b) Give a one-to-one parametrization of $\mathrm{Ker}(A)$.

   (c) Give a one-to-one parametrization of $\mathrm{Im}(A)$.

$\mathrm{rank}(A) \ =$

one-to-one parametrization of $\mathrm{Ker}(A)$

one-to-one parametrization of $\mathrm{Im}(A)$

(Use this page for your calculations)

(Use this page for your calculations)