GEORGIA TECH SCHOOL OF MATHEMATICS

MATH 1502

CALCULUS II

Test # 1 : 50 minutes

September 26th, 2007

First Name : _____

Last Name : _____

1a	
2a 2b	
3a 3b	
4a 4b 4c 4d	
5a 5b	
6a 6b	

WARNING :

Read carefully, read the comments in italic, take your time, do not panic and double check what you write.

Take the time to write in plain English the criteria or the name of the tests you are using to justify your answer.

1. Give the value of $Q^{(11)}(0)$ if

$$Q(x) = 1 - x + 2x^2 - 3x^3 + 4x^5 - 5x^7 + 6x^{11} - 7x^{13}$$

$$Q^{(11)}(0) \ =$$

2. (a) Give the Taylor *expansion* up to order n near $x = 0$ of (*without the remainder*)

$$\frac{1}{1-2x} =$$

(b) Give the Taylor *series*, near $x = 0$ of

$$e^{-3x} =$$

3. (a) Compute the limit (*Give explicitly the rule used to get the result*)

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} =$$

(b) Is the following integral convergent ?

$$\int_0^\infty \frac{dx}{(1+x)^{2/3}}$$

4. Tell whether the following series converge or not and indicate the test used to conclude

   (a)

$$\sum_{n=0}^{\infty} |\cos\{n\pi\sqrt{2}\}|$$

   Converges ☐ *Diverges* ☐

   Test used :

   (b)

$$\sum_{n=0}^{\infty} \frac{111^n}{n!}$$

   Converges ☐ *Diverges* ☐

   Test used :

(c)

$$\sum_{n=0}^{\infty} \frac{1}{(1-n+2n^2+3n^3)^{1.01/3}}$$

Converges ☐ *Diverges* ☐

Test used :

(d)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{1+2n}}$$

Converges ☐ *Diverges* ☐

Test used :

5. Let $f(x)$ be the function given by the power series

$$f(x) \;=\; \sum_{n=0}^{\infty} \frac{x^n}{(1+3n^2)}$$

(a) What is the *radius of convergence* of f ? What test did you use ?

Radius of convergence =

Test used :

(b) Compute the power series expansion of the derivative f' of f

$f'(x) \;=$

6. (a) Compute the following integral

$$I = \int_1^2 \frac{dx}{1+x^2} =$$

(b) Compute numerically the same integral I, by using the *midpoint* method and by dividing the interval of integration into $n = 3$ subintervals

Numerical value :

$$I =$$