GEORGIA TECH SCHOOL OF MATHEMATICS

MATH 1502

CALCULUS II

Test # 2 : 50 minutes

November 3rd 2008, Section D

First Name : ____

Last Name : ____

Students : please do not write anything in the box below!!

1	
2	
3	
4	
5a 5b 5c 5d	
6	
7a 7b	
8a 8b 8c 8d	

WARNING :

Read carefully, read the comments in italic, take your time, do not panic and double check what you write.

Write the result cleanly and use the blank pages for your calculations.

Take the time to write in plain English the arguments used to get or justify the answer.

1. Find a matrix A such that

$$\begin{bmatrix} -2 & 0 \\ 1 & 1 \end{bmatrix} A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} & \end{bmatrix}$$

(Use this page for your calculations)

2. Let $x = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ be two vectors in $\mathbb{R}^3$. Compute the cosine of their angle θ.

$$\cos\theta \; =$$

3. Give the equation of the image of the unit circle by the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$.

4. Find an equation for the plane containing the points $p_0 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $p_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ and $p_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.

Equation $=$

(Use this page for your calculations)

5. Consider the system of linear equations

$$\begin{aligned} x - y &= -1 \\ x + 2y + az &= 0 \\ x - z &= b \end{aligned}$$

(a) For which values of a, b, if any, does this system have a unique solution ?

(b) Then give this solution for any such values of a and b

(c) For which values of a, b, if any, does this system have no solution ?

(d) For which values of a, b, if any, does this system have an infinite number of solution ?

Unique solution $a\,,\; b\; =$

Solution $=$

No solution $a\,,\; b\; =$

$\infty\#$ solutions $a\,,\; b\; =$

(Use this page for your calculations)

6. Compute the inverse (if any) of the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & -1 \\ 3 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

(Use this page for your calculations)

7. Use the Cholesky method to write the following matrix as LL^t and give the expression of L

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3/2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$L = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}$$

Check here that $A = L\,L^t$

(Use this page for your calculations)

(Use this page for your calculations)

8. Consider the lines in $\mathbb{R}^3$ given, parametrically by $\mathbf{x}_1(s) = \mathbf{x}_1 + s\mathbf{v}_1$ and by $\mathbf{x}_2(t) = \mathbf{x}_2 + t\mathbf{v}_2$ with $\mathbf{x}_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$. The goal is to compute the distance between them :

   (a) Find a matrix A and a vector $\mathbf{b}$ so that the least square solution of $A\mathbf{y} = \mathbf{b}$ given the solution.

   (b) Compute the least square solution $\mathbf{y}_0$.

   (c) Compute the distance between the two lines.

$A =$ $\mathbf{b} =$

$\mathbf{y}_0 =$

Distance =

(Use this page for your calculations)

(Use this page for your calculations)