Calculus II, Section K
Quiz # 5
October 1st, 2008

First Name : _____

Last Name : _____

1. Let $f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} (e^x + e^{-x})/2 \\ \cos y \ (e^x - e^{-x})/2 \\ \sin y \ (e^x - e^{-x})/2 \end{bmatrix}$ and let $g(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \begin{bmatrix} y^2 + z^2 \\ x^2 - y^2 - z^2 \end{bmatrix}$.

   Compute $g \circ f$:

$$g \circ f(\begin{bmatrix} x \\ y \end{bmatrix}) \ =$$

2. Let f, g be the two transformations below. For each of them indicate whether it is linear (YES) or not (NO).

$$f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 3x - y \\ -2x + y \end{bmatrix}, \qquad g(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x - y \\ y + x - 1 \end{bmatrix}.$$

$YES \ \square \qquad NO \ \square \qquad\qquad YES \ \square \qquad NO \ \square$

3. Let $A = \begin{bmatrix} 1 & 1 & -1 & 2 \\ -3 & 2 & 14 & -1 \\ 21 & -3 & 7 & 1 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ and let $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. Compute the fourth entry of $A\mathbf{x}$ without computing the whole vector $A\mathbf{x}$.

Result $=$

4. Compute the inverse of the 2×2 matrix $A = \begin{bmatrix} 5 & -2 \\ 2 & -1 \end{bmatrix}$.

$$A^{-1} =$$

5. Let f be the linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$ given first by reflecting about the line $y + x = 0$ and then by reflecting about the line $x = 0$. Compute the matrix A of this transformation :

   Hint : compute the images of the vectors in the canonical basis

$$A = \begin{bmatrix} & \\ & \\ & \\ & \end{bmatrix}$$