

GEORGIA TECH

SCHOOL OF MATHEMATICS

MATH 1502D

CALCULUS II, SECTION D

Test # 1

September 24th, 2008

First Name : -----

Last Name : -----

1a	
1b	
2a	
2b	
3a	
3b	
4a	
4b	
4c	
4d	
5	
6a	
6b	
6c	

WARNING :

Read carefully, read the comments in *italic*, take your time, do not panic and double check what you write.

Take the time to write in plain English the criteria or the names of the tests you are using to justify your answer.

The test will last 50 minutes.

1. (a) Give the Taylor *expansion* of $P(x) = 1 - x + x^3$ near $x = 1$

$$P(x) =$$

- (b) Give the value of $Q^{(15)}(0)$ if

$$Q(x) = 1 - \frac{x^3}{2} + \frac{x^6}{3} - \frac{x^9}{4} + \frac{x^{12}}{5} - \frac{x^{15}}{6} + \frac{x^{18}}{7}$$

$$Q^{(15)}(0) =$$

2. (a) Give the Taylor *expansion*, near $x = 0$, up to order n of (*with the explicit expression of the remainder*)

(*Hint : do not use the general formula for this remainder, use the geometric series.*)

$$\frac{1}{1 + 2x^2} =$$

- (b) Give the Taylor *series*, near $x = 0$ of

$$\frac{\sin \sqrt{2}x}{\sqrt{2}} =$$

3. (a) Compute the limit (*Give explicitly the rule used to get the result*)

$$\lim_{x \rightarrow 0} \frac{x}{e^x - 1} =$$

- (b) Is the following integral convergent ?

(Hint : do not forget the two ends of the interval of integration)

$$\int_0^{\infty} \frac{dx}{(x^3 + x)^{2/3}}$$

4. Tell whether the following series converge or not and indicate the tests used to conclude

(a)

$$\sum_{n=0}^{\infty} \cos^2\{n\sqrt{3}\}$$

Converges

Diverges

Test used :

(b)

$$\sum_{n=0}^{\infty} \frac{33^n}{(n+3)n!}$$

Converges

Diverges

Test used :

(c)

$$\sum_{n=0}^{\infty} \frac{1}{(1 - n + 3n^4)^{1/4}}$$

Converges *Diverges* **Tests used :**

(d)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$$

*Converges absolutely :***YES** **NO** *Converges* *Diverges* **Tests used :**

5. Let $f(x)$ be the function given by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-1)^n}{\sqrt{1+n^2}}$$

What is its *domain of convergence* of f ? Justify your answer

(Hints : beware of the endpoints.)

Domain of convergence =

Tests used :

6. (a) Compute the following integral analytically

$$I = \int_0^1 \frac{dx}{1+x^2} =$$

(b) How can one get the value of $\pi = 3.1415926 \dots$ from this integral.

(c) Compute numerically the same integral I , by using the *trapezoidal* method and by dividing the interval of integration into $n = 2$ subintervals

(Use the back pages for your calculations)

Numerical value :

$$I =$$

(Use this page for your calculations)

(Use this page for your calculations)