

GEORGIA TECH

SCHOOL OF MATHEMATICS

MATH 1502K

CALCULUS II, SECTION K

Test # 1

September 24th, 2008

First Name : -----

Last Name : -----

1a	
1b	
2a	
2b	
3a	
3b	
4a	
4b	
4c	
4d	
5	
6a	
6b	
6c	

**WARNING :**

**Read carefully, read the comments in *italic*, take your time, do not panic and double check what you write.**

**Take the time to write in plain English the criteria or the names of the tests you are using to justify your answer.**

**The test will last 50 minutes.**

1. (a) Give the Taylor *expansion* of  $P(x) = 1 + x^2 - x^3$  near  $x = 1$

$$P(x) =$$

- (b) Give the value of  $Q^{(8)}(0)$  if

$$Q(x) = 1 - \frac{x^2}{2} + \frac{x^4}{3} - \frac{x^6}{5} + \frac{x^8}{8}$$

$$Q^{(8)}(0) =$$

2. (a) Give the Taylor *expansion*, near  $x = 0$ , up to order  $n$  of (*with the explicit expression of the remainder*)

(*Hint : do not use the general formula for this remainder, use the geometric series.*)

$$\frac{1}{1 - x/2} =$$

- (b) Give the Taylor *series*, near  $x = 0$  of

$$\cos(\sqrt{3}x) =$$

3. (a) Compute the limit (*Give explicitly the rule used to get the result*)

$$\lim_{x \rightarrow 0} \frac{2x}{\ln\{(1+x)/(1-x)\}} =$$

- (b) Is the following integral convergent?

*(Hint : do not forget the two ends of the interval of integration)*

$$\int_0^{\infty} \frac{dx}{(x^5 + 3x^6)^{1/5}}$$

4. Tell whether the following series converge or not and indicate the tests used to conclude

(a)

$$\sum_{n=0}^{\infty} (-1)^n \ln\{1+n\}$$

*Converges*    

*Diverges*    

**Test used :**

(b)

$$\sum_{n=0}^{\infty} \frac{29^n}{(3n+2)n!}$$

*Converges*    

*Diverges*    

**Test used :**

(c)

$$\sum_{n=0}^{\infty} \frac{1}{(1 - 2n + 3n^2)^{1/3}}$$

*Converges* *Diverges* **Tests used :**

(d)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\ln\{n + 2\}}$$

*Converges absolutely :***YES** **NO** *Converges* *Diverges* **Tests used :**

5. Let  $f(x)$  be the function given by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(2 - 3x)^n}{(1 + n^3 - n)^{1/3}}$$

What is its *domain of convergence* of  $f$ ? Justify your answer

(Hints : beware of the endpoints.)

Domain of convergence =

Tests used :



6. (a) Compute the following integral analytically

$$I = \int_0^1 \frac{dx}{1+x^2} =$$

(b) How can one get the value of  $\pi = 3.1415926 \dots$  from this integral.

(c) Compute numerically the same integral  $I$ , by using the *middle point* method and by dividing the interval of integration into  $n = 2$  subintervals

*(Use the back pages for your calculations)*

*Data :  $8/17 = 0.470588$*

Numerical value :

$$I =$$

*(Use this page for your calculations)*

*(Use this page for your calculations)*