GEORGIA TECH SCHOOL OF MATHEMATICS

MATH 1502D

CALCULUS II, SECTION D

Test # 2

November 2nd 2009

First Name : _____

Last Name : _____

DO NOT WRITE IN THE TABLE BELOW

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WARNING :

The answer to the first question will be essential for the rest of the problem. Double check your result to make sure that no error was made.

Read carefully, read the comments in italic, take your time, do not panic and double check what you write.

Take the time to write in plain English the criteria or the names of the tests you are using to justify your answer.

The test will last 50 minutes.

The entire test will concern the following matrix

$$A = \begin{bmatrix} 1 & -1 & 1 & 1 & 0 & 5 \\ 1 & -1 & 0 & 1 & 1 & 4 \\ 1 & -1 & 0 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 & 0 & 3 \\ 1 & -1 & 0 & 1 & 0 & 3 \end{bmatrix}$$

1. Row reduce the augmented matrix $[A|I]$ and give U, the row-reduced form, and R the record matrix. *(Do NOT row-echelon reduce!)*
   (Give the result here, compute below and on the back pages)

$U =$

$R =$

(Use this page for your calculations)

2. Give the rank of A,

$$\text{Rank}(A) =$$

3. Give a one-to-one parametrization of $\text{Im}(A)$.

4. Give an equation for $\mathrm{Im}(A)$. *(Hint : use the record matrix in question 1 and find a matrix C such that $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $C\mathbf{b} = 0$)*

5. Does the vector $\mathbf{b}$ below belong to $\mathrm{Im}(A)$?

$$\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \tag{1}$$

6. Give a one-to-one parametrization of $\mathrm{Ker}(A)$.
   (Hint : use back substitution from the reduced form !)

7. By using the expression of A, and the informations coming from the reduced form (see question 1) show that (namely, explain why) the following matrix B satisfies $\mathrm{Im}(A) = \mathrm{Im}(B)$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

8. Check that $B^tB = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ and compute the inverse of B^tB.

   (Hint : use the row reduction method)

   (Give the result here, compute below and on the back pages)

$$\left(B^tB\right)^{-1} =$$

(Use this page for your calculations)

9. Compute the point $\mathbf{b}_0$ in $\mathrm{Im}(A)$ closest to the vector $\mathbf{b}$ in equation (1)
   (Hint : use the least square solution for the equation $B\mathbf{x} = \mathbf{b}$, explain why this is the right method)

$$\mathbf{b}_0 =$$

10. Compute the Cholesky factorization of B^tB, namely compute a lower triangular 3×3 matrix L with positive coefficients on the diagonal, so that $B^tB = LL^t$.

    (Hint : check that $LL^t = B^tB$ after your calculation is done !)

$$L =$$

(Use this page for your calculations)