Georgia Tech School of Mathematics

Math 1502D

Calculus II, Section D

Test # 1

September 23rd, 2009

First Name : ____

Last Name : ____

DO NOT WRITE IN THE TABLE BELOW

1a 1b	
2a 2b 2c	
3	
4a 4b 4c 4d	
5a 5b	
6a 6b 6c	

WARNING :

Read carefully, read the comments in italic, take your time, do not panic and double check what you write.

Take the time to write in plain English the criteria or the names of the tests you are using to justify your answer.

The test will last 50 minutes.

1. (a) Give the Taylor *expansion* of $P(x) = 1 - 4x + 6x^2 - 4x^3 + x^4$ near $x = 1$

$$P(x) \ =$$

(b) Give the value of $Q^{(39)}(0)$ if

$$Q(x) = 1 - \frac{x^{13}}{39} + \frac{x^{26}}{650} - \frac{x^{39}}{37 \times 38 \times 39} + \frac{x^{52}}{24} - \frac{x^{65}}{65 \times 64 \times 63}$$

$$Q^{(39)}(0) \ =$$

2. (a) Give the Taylor *expansion* to order $2n+1$, near $x=0$ of

$$\ln\left\{\frac{1+x}{1-x}\right\} =$$

(b) Compute the numerical value of $\ln(3/2)$ by using $n=1$ and show that this is a lower bound. *(Hint :* $(0.4)/75 = (5+1/3)\times 10^{-3}$*.)*

Put the result here and use the back page for your calculations

(c) Give the corresponding upper bound to $\ln(3/2)$ if one admits that the remainder of the previous expansion is bounded by
(Hint : $1/7500 = 4/3 \times 10^{-4}$*.)* **Put the result here and use the back page for your calculations**

$$0 < R_{2n+1} < \frac{2x^{2n+3}}{(2n+3)(1-x^2)}$$

3. Is the following integral convergent ?
(Hint : identify the points on the interval of integration where the function is unbounded)

$$\int_0^\infty \frac{dx}{(x^5 - x^3 + x)^{3/10}}$$

4. Tell whether the following series converge or not and indicate the tests used to conclude

(a)

$$\sum_{n=0}^{\infty} (-1)^n \left(\frac{n}{3+n}\right)^{n/2}$$

Converges ☐ *Diverges* ☐

Test used :

(b)

$$\sum_{n=0}^{\infty} \frac{89^n}{(n!)^{1/3}}$$

Converges ☐ *Diverges* ☐

Test used :

(c)

$$\sum_{n=0}^{\infty} \frac{\{\ln(n+1)\}^4}{(1+13n^5)^{1/4}}$$

Converges ☐ *Diverges* ☐

Tests used :

(d)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3n+1}}$$

Converges absolutely : **YES** ☐ **NO** ☐

Converges ☐ *Diverges* ☐

Tests used :

5. Let $f(x) = \sum_{n=0}^{\infty} a_n(x-4)^n$ be a power series such that $\sum_{n=0}^{\infty} a_n 3^n$ converges.

   (a) Compute the biggest interval of the form (c, d) in which such a series may converges absolutely.

   Domain of convergence =

   Arguments and Tests used :

   (b) What may happen at the end points ?

   End points :

6. (a) Compute the following integral analytically

$$I = \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$

(b) Give $\pi = 3.1415926\cdots$ in terms of this integral.

(c) Compute numerically the same integral I, by using the *trapezoidal* method with $n = 3$ subintervals and the following data

$$\frac{6}{\sqrt{35}} = 1.014\,, \qquad \frac{6}{\sqrt{32}} = 1.061\,, \qquad \frac{6}{\sqrt{27}} = 1.155$$

(Use the back pages for your calculations)

Numerical value : $I =$

(Use this page for your calculations)

(Use this page for your calculations)

Question 5a)- should be read

"Compute the largest interval of the form (c,d) in which you can guarantee that such a series converges absolutely."