GEORGIA TECH SCHOOL OF MATHEMATICS
MATH 1502

CALCULUS II, SECTION D
Quiz # 5
September 30, 2009

First Name : _____

Last Name : _____

1. Let f, g be the two transformations below. For each of them indicate whether it is linear (YES) or not (NO).

$$f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 3x-5y \\ -4x-2y+1 \end{bmatrix}, \qquad g(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x+y \\ y^3x^2 \end{bmatrix}.$$

YES ☐ *NO* ☐ *YES* ☐ *NO* ☐

2. Let $f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x^2-y^2 \\ 2xy \end{bmatrix}$. Is this map one-to-one ? Justify your answer.

3. Let $A = \begin{bmatrix} 0 & -1 & -1 & -3 \\ 1 & 0 & -4 & 2 \\ 5 & -6 & 3 & 0 \\ 6 & 1 & 0 & -1 \end{bmatrix}$ and let $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$. Compute the third entry of $A\mathbf{x}$ without computing the whole vector $A\mathbf{x}$.

Result =

4. Compute the inverse of the 2×2 matrix $A = \begin{bmatrix} 5 & -\sqrt{3} \\ \sqrt{12} & 1 \end{bmatrix}$.

$$A^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$$

5. Let f be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ given first by a rotation of 45° around the x-axis (anti-clockwise in the plane yz), followed by a rotation of 45° around the z-axis (anti-clockwise in the plane xy) and by the projection parallel to the z-axis onto the plane $z = 0$. Compute the matrix A_f of this transformation :

   Hint : compute the images of the vectors of the canonical basis

$$A_f = \begin{bmatrix} & \\ & \\ & \\ & \\ & \\ & \end{bmatrix}$$