Georgia Tech School of Mathematics

Math 1502D

Calculus II, Section D

Test # 2

November 8th, 2010

First Name : ____

Last Name : ____

DO NOT WRITE IN THE TABLE BELOW

1	
2	
3	
4	
5a 5b 5c 5d 5e	
6a 6b 6c	

WARNING :

Read carefully, read the comments in italic, take your time, do not panic and double check what you write.

Take the time to write in plain English the criteria or the names of the tests or of the objects you are using to justify your answer.

The test will last 50 minutes.

1. Let $f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} \frac{x}{\sqrt{x^2+y^2}} \\ \frac{y}{\sqrt{x^2+y^2}} \end{bmatrix}$ and let $g(\begin{bmatrix} r \\ \theta \end{bmatrix}) = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}$. Compute $f \circ g$:

$$f \circ g(\begin{bmatrix} r \\ \theta \end{bmatrix}) \;=$$

2. If (a, b, c) are three real number let $P(X) = a+bX+cX^2$. Define $Q(X)$ by $Q = \frac{d}{dX}(1-X^2)\frac{dP}{dX}+\lambda P = u+vX+wX^2$. Then compute the matrix A transforming a, b, c into the coefficients u, v, w of the polynomial Q

(Write your answer here and use the back page for your calculations)

$$A \;= \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

(Use this page for your calculations)

3. Write the matrix of the linear transformation $g : \mathbb{R}^2 \to \mathbb{R}^2$ defined as the composition of an anticlockwise rotation of angle $2\pi/5$ followed by a reflection around the line L making the angle $9\pi/20$ with the Ox-axis and $\pi/20$ with the Oy-axis. *(It is advised to draw a picture)*

$$A_g = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

4. Let R be the parallelogram with vertices at $0, \mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_1 + \mathbf{v}_2$, where $\mathbf{v}_1 = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1/2 \\ -1 \end{bmatrix}$. Compute the image of R under the linear transformation of matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

   (Please draw a picture with the coordinates of the vertices of the image)

5. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 2 & -1 \\ 1 & 0 & 2 & -1 & 2 \\ 2 & -1 & 5 & -4 & 5 \end{bmatrix}$.

(a) Through row reduction, find a unit lower triangular matrix L and a reduced matrix U such that $A = LU$

(Write your answer here and use the back page for your calculations)

(Hint : express the record matrix as a block matrix in the form $R = \begin{bmatrix} I & 0 \\ B & I \end{bmatrix}$ *where* I, B *are* 2×2 *matrices ; check that its inverse is* $\begin{bmatrix} I & 0 \\ -B & I \end{bmatrix}$*.)*

$$L = \begin{bmatrix} \quad \end{bmatrix}$$

$$U = \begin{bmatrix} \quad \end{bmatrix}$$

(Use this page for your calculations)

(b) Give an equation for $\mathrm{Im}(A)$

(c) Give a basis for $\mathrm{Im}(A)$

(d) Give a basis for $\mathrm{Ker}(A)$

(e) Compute the distance of the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ -4 \\ -2 \end{bmatrix}$ from $\mathrm{Im}(A)$

(Hint : since $\mathrm{Ker}(A) \neq \{0\}$, *replace* A *by a suitable matrix* B *with* $\mathrm{Im}(A) = \mathrm{Im}(B)$ *and* $\mathrm{Ker}(B) = \{0\}$*)*

$$\mathrm{dist}\{\mathbf{b}, \mathrm{Im}(A)\} =$$

(Use this page for your calculations)

6. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

(a) Compute the inverse of A

(Write your answer here and use the space below and the back page for your calculations)

$$A^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

(Use this page for your calculations)

(b) Show that A is positive definite

(Hint : compute $\mathbf{x} \cdot A\mathbf{x}$ *and write it as a sum of squares, using* $(x-y+z)^2 = x^2+y^2+z^2-2xy+2xz-2yz$*)*

(c) Using the Cholesky method, find a lower triangular matrix L so that $A = LL^t$.

(Write your answer here and use the space below and the back page for your calculations)

$$L = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$$

(Use this page for your calculations)