Georgia Tech School of Mathematics
Math 1502

Calculus II, Section D

Quiz # 5

October 5th 2010

First Name : _____

Last Name : _____

1. Let $f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} \frac{1+x^2-y^2}{x^2+(1+y)^2} \\ \frac{2xy}{x^2+(1+y)^2} \end{bmatrix}$ and let $g(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} \frac{x+y}{\sqrt{2}} \\ \frac{x-y}{\sqrt{2}} \end{bmatrix}$. Compute $g \circ f$:

$$g \circ f(\begin{bmatrix} x \\ y \end{bmatrix}) =$$

2. Let f, g be the two transformations below. For each of them indicate whether it is linear (YES) or not (NO).

$$f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} -\sqrt{3}x + 2y \\ 139x - 3.1415y \end{bmatrix}, \qquad g(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x+1 \\ xy + 3y \end{bmatrix}.$$

YES ☐ *NO* ☐ *YES* ☐ *NO* ☐

3. Write the matrix of the linear transformation f of the form $f\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$ where
$u + vx + wx^2 = -d/dx\,(a + bx + cx^2) + (a + bx + cx^2)$

$$A_f = \begin{bmatrix} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \end{bmatrix}$$

4. Compute the inverse of the 2×2 matrix $B = \begin{bmatrix} 10 & 7 \\ 7 & 5 \end{bmatrix}$.

$$B^{-1} = \begin{bmatrix} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \end{bmatrix}$$

5. Let g be the linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$ given first by an anticlockwise rotation of angle $\pi/3$ followed by a reflection about the line $x + \sqrt{3}y = 0$ *(Hint : this line makes an angle of* $\pi/6$ *with the y-axis ; beware of the slope)*. Compute the matrix A_g of this transformation :

$$A_g = \begin{bmatrix} & \\ & \\ & \\ & \\ & \\ & \\ & \end{bmatrix}$$

(Use this page for your calculations)