CALCULUS II, SECTION K
Quiz # 5
October 5th 2010

First Name : _____

Last Name : _____

1. Let $f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} \frac{2y}{(1+x)^2+y^2} \\ \frac{1-x^2-y^2}{(1+x)^2+y^2} \end{bmatrix}$ and let $g(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} -y \\ x \end{bmatrix}$. Compute $g \circ f$:

$$g \circ f(\begin{bmatrix} x \\ y \end{bmatrix}) =$$

2. Let f, g be the two transformations below. For each of them indicate whether it is linear (YES) or not (NO).
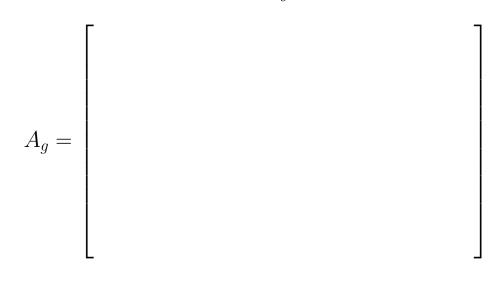
$$f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} \sqrt{5}x + 17y \\ 1.5714x + 67y \end{bmatrix}, \qquad g(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} y - 3x + 2 \\ 2x + xy - y \end{bmatrix}.$$

$YES \ \square \qquad NO \ \square \qquad\qquad\qquad YES \ \square \qquad NO \ \square$

3. Write the matrix of the linear transformation f of the form
$$f\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \text{ where}$$
$u + vx + wx^2 = -2x\,d/dx\,(a + bx + cx^2) + (a + bx + cx^2)$

$$A_f =$$

4. Compute the inverse of the 2×2 matrix $B = \begin{bmatrix} 4 & -3 \\ 7 & -5 \end{bmatrix}$.

$$B^{-1} =$$

5. Let g be the linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$ given first by a clockwise rotation of angle $\pi/3$ followed by a reflection about the line $x - \sqrt{3}y = 0$ *(Hint : this line makes an angle of $\pi/6$ with the y-axis; beware of the slope)*. Compute the matrix A_g of this transformation :

$$A_g = \begin{bmatrix} & & \\ & & \\ & & \\ & & \end{bmatrix}$$

(Use this page for your calculations)