Georgia Tech School of Mathematics

Math 1502K

Calculus II, Section K

Test # 1

September 22nd, 2010

First Name : ____

Last Name : ____

DO NOT WRITE IN THE TABLE BELOW

1a 1b	
2a 2b 2c 2d	
3	
4a 4b 4c 4d	
5a 5b	
6a 6b	

WARNING :

Read carefully, read the comments in italic, take your time, do not panic and double check what you write.

Take the time to write in plain English the criteria or the names of the tests you are using to justify your answer.

The test will last 50 minutes.

1. (a) Give the Taylor *expansion* of $P(x) = 4x - 4x^2 + x^3$ near $x = 2$

$$P(x) \ =$$

(b) Give the value of $Q^{(39)}(0)$ if

$$Q(x) = 1 - \frac{x^{13}}{39} + \frac{x^{19}}{17 \times 18 \times 19} - \frac{x^{39}}{37 \times 38 \times 39} + \frac{x^{52}}{437} - \frac{x^{65}}{65 \times 64 \times 63}$$

$$Q^{(39)}(0) \ =$$

2. The hyperbolic cosine and sine are defined by $\cosh x = \frac{e^x+e^{-x}}{2}$ and $\sinh x = \frac{e^x-e^{-x}}{2}$.

   (a) Compute the first and the second derivative of $\sinh x$

   $$\frac{d \sinh x}{dx} =$$

   $$\frac{d^2 \sinh x}{dx^2} =$$

   (b) Give the Taylor *expansion* to order $2n+2$, near $x = 0$ of $\sinh x$

   Hint : compute first $\cosh 0$ *and* $\sinh 0$

   $$\sinh x =$$

(c) Give the expression R_{2n+2} of the remainder *(Taylor form)* and show that, if $x > 0$, $0 < R_{2n+2} \leq (\cosh x)x^{2n+3}/(2n+3)!$.

(Hint : remark that $\sinh x$ *and* $\cosh x$ *are positive, increasing in* x *for* $x > 0$*)*

Put the result here and use the back page for your calculations

$$R_{2n+2}(x) \ =$$

(d) Gives an interval of values to which the exact value of $\sinh(1)$ belongs, with the help of the previous formula for $n = 1$.
To do so : (i) use the positivity of the remainder to get a lower bound on $\sinh(1)$, *(ii) use the inequality* $\cosh(1) \leq 1+\sinh(1)$ *(without proof) (iii) use the previous upper bound on the remainder, (iv) derive an upper bound for* $\sinh(1)$ *using the previous bounds.*

Put the result here and use the back page for your calculations

$$\leq \sinh(1) \leq$$

(Use this page for your calculations)

3. Is the following integral convergent ?

   (Hint : identify the points on the interval of integration where the integral is improper ; it will be admitted that $x^7 - x^3 + 5 \geq a > 0$ for all $x \geq 0$)

$$\int_0^{\infty} \frac{dx}{(x^8 - x^4 + 5x)^{3/16}}$$

4. Tell whether the following series converge or not and indicate the tests used to conclude

(a)

$$\sum_{n=0}^{\infty}(-1)^n\left(\frac{n}{\sqrt{1+n^2}}\right)^{37}$$

Converges $\square$ *Diverges* $\square$

Test used :

(b)

$$\sum_{n=0}^{\infty}\frac{213^n}{(n!)^{0.001}}$$

Converges $\square$ *Diverges* $\square$

Test used :

(c)

$$\sum_{n=0}^{\infty} \frac{\{\ln(n+51)\}^{111}}{(1+2n^7)^{1/3}}$$

Converges ☐ *Diverges* ☐

Tests used :

(d)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3n+1}}$$

Converges absolutely : **YES** ☐ **NO** ☐

Converges ☐ *Diverges* ☐

Tests used :

5. Let $f(x)$ be the function given by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{\sqrt{1+n+n^2}}$$

(a) Compute the radius of convergence and indicate the test you are using.

Domain of convergence =

Arguments and Tests used :

(b) What will happen at the end points ?

End points :

(Use this page for your calculations)

6. This question concerns the convergence of the following series

$$\sum_{n=1}^{\infty} \frac{n}{1 + 2^2 + 3^2 + \cdots + n^2} \tag{1}$$

   (a) Using a comparison with the function $f(x) = x^2$, show that $1 + 2^2 + 3^2 + \cdots + n^2 \geq \int_0^n x^2 dx$

(b) Using the previous estimate, is the series (1) above converging or diverging ? Give the criterion used to justify your answer.

(Use this page for your calculations)