

GEORGIA TECH

SCHOOL OF MATHEMATICS

MATH 1512

**HONOR CALCULUS II**  
**Preliminary test**  
*July 5th 2005*

*The following consists of few questions that the Mathematics Instructor expects a student following the Honor Calculus II class to know how to solve without the help of friends nor books. Students are strongly encouraged to try to do the short test below within 20 minutes and the problem which follows within 1 or 2 hours. The answers will be posted on the web at a later time in July.*

## 1 A 20-minutes Test

Give the result of<sup>1</sup>

1.

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} =$$

2. Is  $x \in (0, 1) \mapsto \sin\{1/x\}$  continuous at  $x = 0$ ?

3. Is  $\sum_{n=1}^{\infty} 1/n^2$  convergent?

4. Is  $\sum_{n=2}^{\infty} 1/(n \ln n)$  convergent?

5. Compute the derivative of  $\sin^2 x/x^2$

6. Where is the maximum of  $x \mapsto e^{-x^2/2}$ ?

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<sup>1</sup>The answer should be short enough to be written in the space left

7.

$$\int_0^1 dx \, x(1-x) =$$

8. For  $a > 2$ , compute

$$\int_2^a dx \, \frac{1}{x^2 - 1} =$$

## 2 A 1-hour Problem :

1. Solve  $|3 - 2x| < 1$ .
2. Show that  $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - x^2}}{x^2} = \frac{1}{2}$ .
3. Compute the 79-th derivative of  $(2x^8 + 2)^{10}$ . (*Be smart, think before computing!*)
4. Compute  $\int \frac{1}{\sqrt{x^2 + 1}} dx$ .
5. compute  $\int x \sin x dx$ .
6. A function is defined as follows

$$f(x) = \begin{cases} \cos x & x \leq c, \\ ax + b, & x > c, \end{cases}$$

where  $a, b$  and  $c$  are constant. Suppose  $b, c$  are given. Find all possible values of  $a$  (if they exists) such that  $f$  is continuous at  $c$ .

7. Let  $f$  be a continuous function on the interval  $[a, b]$  and differentiable on  $(a, b)$  for some  $0 < a < b$ , such that  $f(a) = f(b) = 0$ . Show that there is  $c$  between  $a$  and  $b$ , such that  $f'(c) = f(c)/c$ . Draw a picture to illustrate this result.