Georgia Tech School of Mathematics

Math 1512

Honor Calculus II

1/2-hour Test

September 22nd, 2004

First Name : ____

Last Name : ____

1. Give the Taylor *series* of

$$\sin(x) \ :$$

$$\frac{1}{\sqrt{1-x}} \ = \ (1-x)^{-1/2} \ :$$

2. Give the Taylor *expansion* up to order n of (with the **explicit expression of the remainder**)

$$\frac{1}{1-x^2} \ =$$

3. Compute

(a)

$$\lim_{n\to\infty} (n+a)^{1/3}(n+b)^{2/3}-n \ =$$

(b)

$$\lim_{n\to\infty} \left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right) \ =$$

(c) for $a > 1$ and $\alpha > 1$

$$\lim_{n\to\infty} \frac{a^{n^\alpha}}{n!} \ =$$

(d) Is the following series convergent and why ?

$$\sum_{n>2} \frac{1}{n \ln^{3/2} n} \ :$$

(e) Is the following series convergent and why ?

$$\frac{1}{\ln(2)} - \frac{1}{\ln(3)} + \frac{1}{\ln(4)} + \cdots + (-1)^n \frac{1}{\ln(n)} + \cdots$$

4. (a) Shows that the integral $\int_0^\infty dy\, e^y/y^y$ converges (*Hint : compare with the series* $\sum_{n=1}^\infty (e/n)^n$)

   (b) Using the integral test, is this series convergent or not ?

$$\sum_{n=1}^{\infty} \frac{1}{(\ln n)^{\ln n}} :$$

5. Show that the following series converges uniformly on $\mathbb{R}$

$$\sum_{n=1}^{\infty} \frac{\sin(3^n x)}{2^n} :$$