Georgia Tech School of Mathematics
Math 1512

Honors Calculus II

1-hour Test # 1

September 28th, 2005

First Name : ____

Last Name : ____

WARNING : read carefully, read the comments in italic, take your time, do not panic and double check what you write.
Take the time to write in plain English the criterion or the name of the test you are using to justify your answer.

1	
2a 2b	
3a 3b 3c	
4a 4b	
5a 5b 5c 5d	
6a 6b	

1. Give the Taylor *expansion* up to order n of (*with the explicit expression of the remainder*)

$$\frac{1}{1-x} =$$

2. Give the Taylor *polynomial* up to order n of (*no remainder needed, but justification must be explicit*)

$$\frac{1}{1+x^2} \ :$$

$$\tan^{-1} x \ :$$

3. Compute

(a)

$$\lim_{n\to\infty} n - \sqrt{n+a}\sqrt{n+b} \ =$$

(b)

$$\lim_{n\to\infty} \left(\frac{n}{n^2+1} + \frac{n}{n^2+2^2} + \cdots + \frac{n}{n^2+n^2} \right) \ =$$

(c) for $a > 1$

$$\lim_{n\to\infty} \frac{a^{n^2}}{n^n} \ =$$

4. (a) Is the following series convergent and why ?

$$\sum_{n\geq 2} \frac{1}{n \ln^2 n} \quad :$$

(b) Is the following series convergent and why ?

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots + (-1)^{n-1}\frac{1}{\sqrt{n}} + \cdots$$

5. For any real number α and any natural integer n let the *binomial cœfficient* be defined as

$$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!} \qquad \text{if } n \neq 0 \text{ and } \qquad \binom{\alpha}{0} = 1\,.$$

In this problem, $f(x) = (1+x)^\alpha$ for $x > -1$.

(a) Compute the first derivative of the function f.

$$f'(x) \; =$$

(b) Using recursion, compute $f^{(n)}/n!$.

$$\frac{f^{(n)}(x)}{n!} \; =$$

(c) Gives the Taylor *series* of f at $x = 0$ (*give the full power series expansion*)

$$f(x) \;=$$

(d) What is the radius of convergence for this series (*justify your result in plain English*) ?

Interval of convergence =

Justification :

6. For the following sequence $\{f_n\}$ of functions, determine the pointwise limit and decide whether it converges uniformly or not. (*Hint : be careful at values of x for which $f_n(x)$ does not depend on n*)

   (a)

$$f_n(x) \; = \; e^{-nx^2} \qquad \text{on} \quad [-1, +1]$$

   (b)

$$f_n(x) \; = \; \sqrt{x + \frac{1}{n}} - \sqrt{x} \qquad \text{on} \quad [0, +\infty)$$