GEORGIA TECH SCHOOL OF MATHEMATICS
MATH 2401

# CALCULUS III
**Test # 1**
*September 20th, 2012*

**First Name :** ______________________________________________

**Last Name :** __________________________________________________

## DO NOT WRITE IN THE TABLE BELOW

| | |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5a<br>5b | |
| 6a<br>6b | |
| 7a<br>7b | |
| 8a<br>8b | |
| 9a<br>9b<br>9c | |

**WARNING :**

**Read carefully, read the comments in italic, take your time, do not panic and double check what you write.**

**Write CLEARLY your answer where it is asked to.**

**The problems 4 and 5 are the most time consuming.**

**The test will last 50 minutes.**

1. Calculate the derivative of $\vec{f}(t) = \cos 2t\vec{i} + e^{-t} sin3t\vec{j} + (t^2 + 2t)\vec{k}$

$$\vec{f'}(t) \; =$$

2. Calculate the derivative of $\vec{g}(t) = (e^{-t}\sin t\vec{i} + \ln(1+t^2)\vec{j}) \times (3t^2\vec{i} + t^3\vec{j} + e^{-t}\vec{k})$

$$\frac{d\vec{g}}{dt} \; =$$

3. Find the unit tangent of $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$

$$\vec{T}(t) \ =$$

4. Find the length of the curve $\vec{r}(t) = 3t\cos t\ \vec{i} + 3t\sin t\ \vec{j} + 4t\ \vec{k}$ from $t = 0$ to $t = 4$
   *(Hint : if* $\sinh^{-1}(12/5) = \ln(5)$*)*

$$\ell \ =$$

5. Find the coordinates $(x_M, y_M)$ of the maximum of $y = 3x - x^3$ and compute the curvature $\kappa_M$ of the graph at this point.

   *(Hint : (i) compute the position of the local maximum (ii) find a parametric representation, for instance with the parameter $t = x$ (iii) derive the expression of the curvature in term of $t$ (iv) compute the velocity and the acceleration vectors at the maximum)*

Maximum

$$(x_M, y_M) \ =$$

$$\kappa_M \ =$$

6. Find the domain and the range of the function

$$f(x, y, z) = -\frac{z^2}{\sqrt{x^2 - y^2}}$$

Range :

Domain

7. Identify the surfaces :

$$9x^2 + 4y^2 - 36z = 0$$

$$9x^2 + 4y^2 - 36z^2 = 1$$

8. Identify the level curve of

$$f(x,y) = \frac{x^2}{x^2+y^2}\,, \qquad \text{at} \qquad c = \frac{1}{4}$$

and sketch it.

NAME *of the level curve* :

SKETCH

9. Compute the partial derivatives of $f(x, y, z) = z \arctan(y/x)$

$$\frac{\partial f}{\partial x} =$$

$$\frac{\partial f}{\partial y} =$$

$$\frac{\partial f}{\partial z} =$$