Volume 13

CRM MONOGRAPH SERIES

Centre de Recherches Mathématiques
Université de Montréal

Directions in Mathematical Quasicrystals

Michael Baake
Robert V. Moody
Editors

The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FCAR), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal, whose constituent members are Concordia University, McGill University, the Université de Montréal, the Université du Québec à Montréal, and the Ecole Polytechnique. The CRM may be reached on the Web at www.crm.umontreal.ca.

American Mathematical Society
Providence, Rhode Island USA

Contents

Preface

The experimental discovery of real-world quasicrystalline materials by several groups in the early 1980's brought into question a number of long-standing assumptions in crystallography regarding the relationship between long-range order and periodic order. By now it is clear that there is indeed a very real world of aperiodic long-range order, and there are many fascinating questions to ask and to answer about the vast territory that lies between crystallographic order and random (or amorphous) (dis)order.

Once the dust had settled a bit, after the initial outburst of activities in the world of physics, it turned out that, on the one hand, there were a number of important mathematical predecessors (some of which were painfully reestablished), and that, on the other hand, many new questions emerged that needed (and still need) professional mathematical attention.

The latter started with a NATO ASI workshop at the Fields Institute in fall 1995. Two books with research articles grew out of that: *The mathematics of long-range aperiodic order* (R. V. Moody, ed.), NATO ASI Ser. C: Math. Phys. Sci., vol. 489, 1997, and *Quasicrystals and discrete geometry* (J. Patera, ed.), Fields Inst. Monogr., vol. 10, 1998.

In spring 1998, a meeting on aperiodic order took place in Oberwolfach, and it was the stimulating atmosphere of this meeting together with the large number of new results that led to the decision to edit this volume. It is not meant as a proceedings of this meeting, and in fact it isn't. The reader will find a number of contributions that emerged from talks given at that meeting, as well as a number of additional ones that we have solicited.

All articles were especially written for this volume, with the aim of giving an account of our present knowledge and of the open questions of the field (or at least a substantial part of it). Taken together with the other two volumes, we hope that a rather coherent picture will emerge and that this will serve as a guide and inspiration to those interested in learning more about the mathematics of aperiodic order.

We would like to express our gratitude to our contributors for taking the time and care to produce these accounts of their work and that of their fellow coworkers. Particular thanks also go to Uwe Grimm and Moritz Höffe for providing several figures and for helping us in editing some of the contributions. We also thank our

referees for their competent and timely responses, as well as André Montpetit from CRM and the publisher for excellent cooperation.

Tübingen and Edmonton,
March 2000

Michael Baake and Robert V. Moody