Georgia Tech , Schools of Math & Physics
MATH 8823 BED, Random Matrix Theory

Random Matrix Theory

Final Exam

Three subjects are proposed below. One among them will be assigned to each candidate. Each candidate will be asked to provide a written report (handwritten, printed or LaTeX file) on the assigned subject by **December 6th, 2002**. The report should contain:

1. a presentation of the topic,
2. a short history,
3. a comprehensive bibliography,
4. the list of the main results,
5. at least one example of calculation,
6. the proof of at least one important mathematical result,
7. at least one important application to physics together with the description of the corresponding (theoretical and experimental) results,
8. a conclusion.

The *length of this report should not be more than 10 LaTeXor 20 handwritten pages.* The evaluation of the work will take into account both the content and the quality of the text and the presentation. A guideline is proposed for each topic. The candidates can find a short list of references to start with on

`http://www.math.gatech.edu/~jeanbel/` (go to `Teaching`)

1)- The Gaussian Orthogonal Ensemble [1] :

Definition of the Gaussian Orthogonal Ensemble. Joint probability distribution of the eigenvalues. The method of orthogonal polynomials. Density of states and the 2-points correlation function: results and computation. Level spacing distribution: definition, computation (Gaudin's method).

2)- Voiculescu's Entropy [2] :

Definition of the Voiculescu *free entropy*. The case of one random variable. Density of states of the distribution maximizing the entropy in the following two cases:

- the density of states is supported by the interval $[-a, +a]$ for $a > 0$;
- the p-th moment $\int d\mu(x)|x|^p$ is bounded by some fixed constant.

Main properties of the free entropy of a N-tuple $(a_1, \cdots, a_N)$ of quantum random variables: subbadditivity, upper semicontinuity, change of variables, additivity of free entropy. Characterization of freeness from the maximum free entropy principle.

3)- Supersymmetric Method [3] :

Definition of Grassmann variables. The rules for Calculus: algebra, integrals, gaussian integrals. Computation of the density of states of the GUE by using the supersymmetric calculus [4]. Computation of the 2-points correlation function. One problem in mesoscopic physics that can be treated in this way. The choice of the application is left to the candidate.

References

[1] M. Mehta, *Random Matrices*, 2nd Ed., Acad. Press, (1990),

[2] F. Hiai, D. Petz, *The Semicircle Law, Free Random Variables and Entropy*, American Mathematical Society, (2000).

[3] K. B. Efetov, *Supersymmetry in Disorder and Chaos*, Cambridge University Press, (1997).

[4] A. Mirlin, *Statistics of energy levels and eigenfunctions in disordered and chaotic systems: Supersymmetry approach*, *Phys. Rep.*, 326, (2000), 259-382.